Why Existence Cannot Ground Itself

Gödelian Incompleteness and the Structure of Physical Law

Geir Isene

isene.com

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Abstract

Existence exhibits formal structure. Physical laws hold, mathematics applies, and the results stay consistent across every domain we have tested. I argue that this structure cannot account for itself. Gödel's second incompleteness theorem forbids any consistent formal system containing arithmetic from proving its own consistency, and the lawful structure of existence either is such a system or produces one. On the first horn the theorem applies directly. On the second, the unexplained appearance of formal structure from a non-formal substrate demands its own account. Either way the grounding of existence lies outside existence.

I call this requirement *trans-existential grounding*. Three further routes reach it without Gödel: the Leibnizian route through sufficient reason, the property regress, and the exhaustive why. I examine what they owe each other, and conclude that three of the four are genuinely separate starting points while the exhaustive why is sufficient reason carried out as a procedure. Beyond independence they supply a composite description, each route fixing a different constraint on what the ground must be. I then show that the standard candidates fail, and for a common reason. Each is either another part of what needs grounding, or possesses determinate properties that reintroduce the question it was invoked to settle. What survives is a set of constraints rather than a positive identification. Naming the ground is a further question, and I do not attempt it here.

1 The claim

Take existence to be everything that has properties: physical objects and the laws they obey, and also thoughts, concepts, numbers, and proofs. Nothing in what follows depends on drawing the boundary of that set precisely. It depends only on the observation that whatever falls inside it has properties and exhibits structure, and that the structure holds together.

My claim is that this structure cannot be self-supporting.

The argument is short. Existence displays lawful, consistent, mathematically expressible order. Order of that kind constitutes a formal system, or at minimum generates one. Gödel showed in 1931 that a consistent formal system containing arithmetic cannot establish its own consistency from within. So the consistency we observe is not something existence can supply for itself. Something outside it does the supplying.

I call the requirement *trans-existential grounding*, and I use the term throughout: existence is grounded by something that is not itself existent. The name is deliberately thin. It records a requirement and commits to no candidate, and one purpose of this paper is to show how much can be established at that level of thinness. A framework built on the requirement, which does identify a candidate, is developed in a companion paper [2].

That conclusion sounds larger than it is, and part of my purpose is to make clear how modest it actually is. It does not tell us what the external ground is. It does not smuggle in a deity, a mind, or a cosmic purpose. It constrains rather than describes: whatever grounds existence cannot be one more thing inside existence, and cannot have the kind of determinate character that would put it there. It cannot have any specific properties.

2 Existence has formal structure

Start with what is not in dispute. Physical law is expressed mathematically and the expression works. Newtonian mechanics gives arithmetic relations between mass, force and acceleration. General relativity gives differential geometry. Quantum mechanics gives operators on Hilbert spaces. In each case the mathematics is not decoration laid over the physics afterwards; it is how the regularity is stated, and the predictions come out right to as many decimal places as we care to measure.

Wigner called this the unreasonable effectiveness of mathematics and declined to explain it [25]. His puzzlement is the beginning of my argument. Formal structure in existence is not an artefact of how we happen to describe things. Descriptions can be replaced. The regularities they track do not go away when the notation changes.

Definition 1 (Existence)**.** Let $\mathcal{E}$ denote the totality of what has properties: physical entities and their laws, mental contents, mathematical objects, and formal systems.

The definition is deliberately generous. A narrower one, restricted to the physical, would leave mathematics outside $\mathcal{E}$ and invite the reply that mathematical structure is grounded Platonically and the problem dissolves. It does not dissolve; it relocates, and I address that relocation in Section 5. Taking $\mathcal{E}$ to include everything with properties closes that exit at the start.

3 The structural argument

Two premises and a theorem.

The first premise is that the lawful structure of $\mathcal{E}$ constitutes a formal system containing arithmetic. Physical laws rest on axioms, relate quantities by definite rules, and yield arithmetic relations among those quantities. Whatever else that is, it satisfies the conditions Gödel's theorems require.

The premise needs support, since some mathematical theories are complete. Tarski proved this for the theory of real numbers under addition and multiplication, and Euclidean geometry is complete too. A physical theory whose mathematics sat inside one of those would be untouched by incompleteness. The mathematics of physics does not sit there. The wave equation carries computable starting data to values that cannot be computed [20]. Whether a given set of tiles covers the plane has no general answer [3]. The local interactions of a quantum lattice can encode a universal Turing machine, which leaves a basic question about its energy spectrum undecidable in the infinite-lattice limit [6]. Mathematics that can carry universal computation can carry arithmetic, and arithmetic is what the theorems require.

The second premise is that $\mathcal{E}$ is consistent. This is an empirical claim and it could in principle fail, but every test we have run supports it. Physics does not deliver contradictory predictions in domains where it has been checked. If existence were inconsistent, arbitrary conclusions would follow from its laws, and they do not.

Theorem 1 (The grounding requirement)**.** *If the lawful structure of $\mathcal{E}$ is a consistent formal system containing arithmetic, then $\mathcal{E}$ cannot establish its own consistency, and any account of that consistency must appeal to something outside $\mathcal{E}$.*

Proof. By Gödel's second incompleteness theorem, a consistent formal system S containing arithmetic cannot prove $\mathrm{Con}(S)$ within S [13]. Let S be the lawful structure of $\mathcal{E}$. By premise one, S satisfies the theorem's conditions; by premise two, S is consistent. So $\mathrm{Con}(S)$ is not derivable in S. Any derivation of $\mathrm{Con}(S)$ requires a system strictly stronger than S. Every candidate within $\mathcal{E}$ is a subsystem of S or equal to it, since S was taken to be the whole lawful structure of $\mathcal{E}$. So no such candidate suffices, and the account must appeal to what lies outside $\mathcal{E}$. □

A word about what the theorem does and does not say. It does not say existence is inconsistent, nor that its consistency is unknowable, nor that some law remains undiscovered. The gap is not epistemic. No possible complete set of laws internal to $\mathcal{E}$ can close it, because the obstacle is structural and Gödel proved it exhaustively for systems of this kind.

3.1 The disjunctive form

The natural objection is that $\mathcal{E}$ merely *contains* formal systems without *being* one. Mathematics is inside existence; existence itself might be something else entirely, something on which Gödel's theorems have no hold.

Grant the objection and the argument survives, by a different route.

If $\mathcal{E}$ is a formal system, Theorem 1 applies as stated. If $\mathcal{E}$ is not a formal system, then $\mathcal{E}$ is a non-formal substrate that nonetheless produces formal structure: arithmetic, logic, consistent and mathematically exact physical law. That production now needs an account. A substrate with no formal character has generated formal character, reliably, at every scale we examine. Calling the substrate non-formal renames the difficulty rather than removing it, and the renaming makes matters worse, since we now have both the structure to explain and its emergence from something with no structure to give.

So the disjunction closes. Whether $\mathcal{E}$ is formal or produces the formal, its order is not self-accounting.

3.2 The pattern behind the theorem

Gödel's result is one instance of a wider limitation. Cantor showed no enumeration captures all reals. Tarski showed no language defines its own truth predicate [22]. Turing showed no procedure decides whether an arbitrary program halts [24]. Lawvere later proved these are the same categorical fact [16].

One might hope to escape Gödel by ascending: work in a richer theory, a stronger set theory, category theory itself. Lawvere's result closes that hope. The limitation is not a defect of one formalism that a better formalism repairs. It reappears at every level of abstraction because it follows from self-reference, and self-reference is what a system grounding itself would require.

4 Three further routes, and what each adds

An argument resting on a single theorem inherits that theorem's contested applications. Mine does not rest on one. Three further routes arrive in the same territory, and I name them here because the rest of the paper refers back to them: the *Leibnizian route* through sufficient reason, the *property regress*, and the *exhaustive why*.

A word first about what their agreement is worth, since the obvious claim to make is stronger than the one I am entitled to. It would be convenient to say that four independent arguments converge on one conclusion, and that rejecting any one leaves three standing. I do not think that survives inspection, and Section 4.4 says why. What the routes actually supply is a composite description. Each fixes a different constraint on the ground, and the interesting fact is that the constraints turn out to be jointly satisfiable rather than that they are separately provable.

4.1 The Leibnizian route: sufficient reason

Leibniz held that everything which exists has a reason for existing. Apply this to $\mathcal{E}$. Our universe is contingent in the ordinary sense: the constants could have taken other values, the laws could have had another form, and nothing about them announces itself as the only possibility. Contingent things want reasons. A reason drawn from inside $\mathcal{E}$ is itself contingent and demands a reason, so the chain does not terminate inside. It terminates only in something necessary, and necessity is precisely what nothing inside $\mathcal{E}$ possesses.

The standard escape is to deny the principle. One may simply refuse to accept that contingency requires explanation. I note only that the refusal is available at any point in any inquiry, that we decline to use it everywhere else in science, and that the cost of using it here is giving up on the question rather than answering it.

4.2 The property regress

Everything in $\mathcal{E}$ has properties. Complex properties reduce to simpler ones: the hardness of a diamond to its lattice, the lattice to interatomic forces, the forces to a handful of constants and symmetries. The reduction is one of the great achievements of physics and it does not go on forever. It bottoms out in fundamental properties: the charge on the electron, the fine structure constant, the dimensionality of spacetime.

Ask why those. There is no further physics to appeal to, since these are the terms in which the physics is stated. Any answer that cites another property has not terminated the regress, it has extended it.

The descent is still being carried out, which makes the point easier to see than it would have been a century ago. Emergent-gravity programmes derive the gravitational field equations from thermodynamic or information-theoretic quantities rather than positing them [14, 4], so a property once taken as fundamental turns out to rest on something further down. Each such result moves the floor. None of them has yet reached one, because what is found underneath is in every case more structure: a functional, a geometry, a set of equations. That is the regress continuing.

It matters that this can be run without any principle of explanation. The reduction is a dependency relation, not a request: the hardness of the diamond obtains *in virtue of* the lattice, whether or not anyone asks after it. Read that way the descent is a grounding regress of the kind studied independently of sufficient reason [21, 8], and the question of whether such chains must be well-founded is live on its own terms [5].

So posed, the regress admits exactly three exits.

1. It terminates in something with no properties, which is outside $\mathcal{E}$ by Definition 1.

2. It descends forever, each property grounded in a further property without end.

3. It stops at properties that are simply had, grounded in nothing.

Option two is metaphysical infinitism, and it faces the difficulty every infinite regress faces: an unending chain of dependencies never supplies what the chain was invoked to supply, since each link borrows what it has yet to receive. Option three is brute facticity relocated to the level of properties, and Section 5 treats it there. Option one is the conclusion I want, and I note that I reach it here by elimination among three structural possibilities rather than by insisting that properties call for explanation.

This route therefore reaches the destination without Gödel and without Leibniz, and it sharpens the destination, since it tells us the ground must be property-free rather than merely external.

4.3 The exhaustive why

The third route is the least formal and the hardest to evade. Take any feature of existence and ask why it holds. Each answer cites a prior condition or a deeper law. Ask again. The chain runs backward through cosmology to initial conditions, to the laws that make them possible, to whatever settles their form.

Three terminations are available. The chain runs forever, which answers nothing, since an infinite series of explanations each borrowing from the next explains no more than a chain hanging from nothing. Or it stops at a brute fact, which is a decision to stop rather than an answer. Or it terminates in something that needs no why, and nothing inside $\mathcal{E}$ qualifies, because everything inside has features that could have been otherwise.

4.4 What the routes owe each other

Four routes, then: incompleteness, sufficient reason, property regress, exhaustive why. The temptation is to present them as four independent supports, so that cutting any one leaves the argument standing on three. That would be a stronger claim than I can defend, and it is worth saying why before a reader says it for me.

One of the four does lean on another. Run the exhaustive why without assuming that things have reasons and the chain of questions still terminates, but the termination carries no weight, since "the questions ran out" and "we reached something that needs no question" become indistinguishable. What makes the terminus significant is the prior conviction that features of the world call for explanation. The exhaustive why is therefore the Leibnizian route carried out as a procedure rather than asserted as a principle, and I do not count it separately.

The property regress is a different case, and I want to be careful not to concede it too readily. It might look like a sufficient-reason demand in other clothes, on the grounds that asking why *these* properties is asking for a reason. But Section 4.2 sets it out as a dependency structure rather than a question: properties obtain in virtue of other properties, and that relation holds whether or not anyone interrogates it. The trilemma follows from the structure alone. Either the chain bottoms out in something property-free, or it descends forever, or it stops arbitrarily. No principle of explanation is needed to generate those three options, and grounding regresses are studied on exactly these terms without any appeal to sufficient reason [21, 5].

That leaves three genuinely separate starting points rather than four or two: Gödel's theorem, which needs no principle of explanation; the property regress, which needs a dependency relation but no principle; and the demand for explanation itself, which appears in two presentations.

Three separate starting points is a weaker claim than four and a stronger one than the sceptic will expect, and it has a consequence worth stating plainly. Someone who rejects the application of incompleteness to existence, which is the most contested step in this paper, still faces the property regress, which uses no theorem, and still faces the demand for explanation. The argument does not stand or fall with Gödel.

Beyond independence, each route fixes a different constraint on the ground:

- Gödel says the ground must be **outside** the system it grounds.
- Sufficient reason says it must be **necessary**, since a contingent ground reopens the question.
- The property regress says it must be **property-free**, since properties are what the regress descends through.
- The exhaustive why says it must **need no further why**, which is what terminating a question means.

Read that way the routes are complementary as well as partly independent. The result worth noticing is that four constraints derived by different means turn out to be jointly satisfiable by one kind of thing, and Section 5 is the search for what kind.

5 Why the regress does not terminate inside existence

Suppose one accepts that a ground is required and looks for it. Any candidate must meet four conditions, and the third is where most proposals fail.

Transcendence. The ground must lie outside $\mathcal{E}$. A ground inside $\mathcal{E}$ is part of what requires grounding, and the question simply recurs.

Productive capacity. It must be able to give rise to what it grounds. A ground that cannot produce anything grounds nothing.

Absence of determinate properties. Anything with determinate properties invites the question why those properties rather than others, and by Definition 1 anything with properties belongs to $\mathcal{E}$. A ground characterised by definite features has been placed inside the set it was meant to support.

Capacity to sustain consistency. The order in $\mathcal{E}$ persists. Whatever accounts for it must account for its persistence, not merely its origin.

5.1 A deity with attributes

Classical theism satisfies transcendence, productive capacity and the sustaining of order. It fails on properties.

The God of that tradition is wise, good, loving, and personal. Each attribute is determinate, and each invites the question the ground was introduced to end. Why wise rather than indifferent? Why loving rather than cruel? These are not impious questions, they are the same question we asked about the fine structure constant, and at the same level. A being with a definite character has a character that could have been otherwise.

The theist may answer that the attributes are essential rather than contingent, that God could not have been otherwise. This shifts the difficulty into the notion of essence without dissolving it, since we may now ask why this essence is the one that is instantiated. Alternatively the theist may strip the attributes away, following the negative theology of Pseudo-Dionysius or Eckhart toward a godhead beyond predication. That move satisfies the property condition. It also abandons the deity of classical theism, which was the candidate under consideration.

5.2 Platonic forms

Abstract objects satisfy transcendence, if one takes them to be genuinely outside space and time, and they satisfy consistency handsomely, since mathematical truths do not contradict each other. They still have defining properties. And they fail productive capacity.

On the standard conception abstract objects are causally inert. This is not an incidental feature; it is close to what makes them abstract. The number seven does not do anything. The Pythagorean theorem brings about no states of affairs. Whatever grounds existence must actualise, and a realm of eternal truths sits there being true, which is a different accomplishment.

Some hold that mathematical structure does not merely describe reality but constitutes it, as Tegmark proposes [23]. That view rescues productive capacity by identifying $\mathcal{E}$ with mathematical structure. It also brings the whole problem back inside, since mathematical structure is formal structure, and Theorem 1 applies to it directly.

5.3 The quantum vacuum

This candidate fails transcendence outright, and the failure is instructive because the proposal is often presented as though it succeeded.

The vacuum is a state of quantised fields. It has an energy density, obeys the equations of quantum field theory, and possesses definite symmetry properties. Every one of those is a property, and every one places it inside $\mathcal{E}$. Krauss's account of a universe arising from nothing describes fields in a particular state and calls the state nothing [15]. Albert's response is difficult to improve on: relativistic quantum field theories are theories of how fields behave, not theories of how fields come from nothing [1].

The general form of the mistake is worth naming, since it recurs. A physically minimal condition is described, then the word "nothing" is applied to it, and then the explanatory work is done by the word rather than the physics.

5.4 The multiverse

An ensemble of universes fails transcendence for the same reason a single universe does, at larger scale.

If one universe with lawful structure requires an account, an infinity of them requires infinitely many accounts, or one account covering the ensemble. The ensemble itself has a character: a measure over possibilities, a generating mechanism, a rule determining which universes are realised. Those are properties, the ensemble is inside $\mathcal{E}$.

The multiverse does useful work against fine-tuning arguments, where the point is to remove the appearance of design by supplying many trials. It does no work against the grounding requirement, because supplying more of the thing to be explained is not an explanation of it.

5.5 Emergence

Emergent properties arise from the organisation of a system's parts. Wetness from molecules, temperature from motion, perhaps consciousness from neurons. The concept is respectable and it does not apply here.

Emergence within $\mathcal{E}$ presupposes $\mathcal{E}$. It presupposes parts, relations between them, and laws governing the relations, which is exactly the structure whose grounding is in question. To say existence emerges is to say the parts and their laws produced themselves, which is the position we started from. Weak emergence is a claim about explanatory levels inside a system; strong emergence, whatever its merits, is a claim about novel causal powers appearing at higher levels of a system. Neither is a claim about the system's ground.

5.6 A necessary being

The classical answer to contingency is something that exists necessarily. Plantinga's modal ontological argument develops this rigorously, concluding that a maximally great being exists in every possible world [19].

The structural parallel with my argument is close, and the divergence is exactly the property condition. A maximally great being has maximal properties: omniscience, omnipotence, perfect goodness. Ask why greatness consists in those and the regress resumes. Necessity itself is a modal property, and a being whose necessity is a feature of it is a being with a feature.

I take Plantinga's argument to establish something real, which is that contingent existence requires a necessary termination. I take it to misidentify the termination by giving it a character.

5.7 Brute fact

The last candidate declines the question. Existence is, no reason, nothing further to say.

This is always available and it never informs. Its cost is not that it is false, since it may be, but that it is the one move which guarantees no progress. It is the capitulation of inquiry we refuse everywhere else. No cosmologist accepts brute facts about the cosmic microwave background. No physicist accepts it about the value of a coupling constant. Reserving it for the largest question is a preference about where to stop asking.

5.8 The pattern in the failures

A single fault runs through all of them. Every candidate is either another inhabitant of $\mathcal{E}$, in which case it inherits the entire problem, or it carries determinate character, in which case Definition 1 puts it back inside $\mathcal{E}$ and it inherits the problem again.

The failures mark the shape of the constraint.

6 What survives

The eliminations leave conditions rather than an identification. An adequate ground is outside $\mathcal{E}$, has no determinate properties, can nonetheless give rise to what has properties, and can sustain the resulting order. Three of those four are the constraints of Section 4.4 arriving from different directions, which is the composite description promised there. The fourth, productive capacity, comes from the elimination itself, since a ground that produces nothing was never a candidate.

Those conditions are jointly restrictive and they are not obviously satisfiable. I have argued that existence requires something whose description approaches a description of nothing at all. A ground with no properties is not a thing among things, and calling it a thing may already be a mistake of grammar.

Whether anything answers to the description, and what it would be, is a further question and a harder one. I take it up elsewhere. Here I stop at the negative result, because it stands on its own and does not depend on how the further question is settled.

7 Related work

The application of Gödel's theorems outside mathematics has a contested history, and my argument needs to be located within it precisely, since the objections to its predecessors are well known and mostly deserved.

Lucas and Penrose. Lucas argued that Gödel's theorems show the human mind exceeds any machine, since we can see the truth of a Gödel sentence the machine cannot prove [17]. Penrose developed a version resting on quantum gravitational processes in neural microtubules [18]. Both claims are about minds outperforming formal systems.

My argument makes no claim of that kind. Nothing here says human cognition escapes any formal limit, and nothing depends on our capacity to see truths a machine cannot. The claim is that formal structure within $\mathcal{E}$ requires grounding from outside $\mathcal{E}$, which is a claim about systems, not about people. The distinction matters because the standard objections to Lucas and Penrose target the inference from incompleteness to human cognitive superiority, and that inference does not appear here.

It also puts me at odds with Penrose on a point of location. He places non-computability inside physics. Whatever its physical realisation, a quantum gravitational process is a process in $\mathcal{E}$, governed by law, and so falls under the argument of Section 3 like anything else.

Feferman. Feferman's critique of the Gödelian arguments for mind is careful and, in my view, decisive against its targets [9]. He shows that the inference from "formal systems are incomplete" to "minds transcend formal systems" blurs which system is at issue and what the knower is assumed to know. Since I draw no conclusion about minds, the critique is not relevant to this article.

Franzén. Franzén's book is the standard treatment of Gödelian overreach, and Section 8 addresses it directly [10].

Gisin. Gisin has argued over the last several years that classical physics is not in fact deterministic, on the ground that real numbers carry infinite information which no finite region can hold [11, 12]. His conclusion is that the future is genuinely open and that propensities are physically real. I arrive near his position from the opposite direction, mathematics rather than physics, and the two arguments are independent. It matters here because it removes a background assumption my argument would otherwise have to fight, namely that a fully determined block universe leaves nothing for a ground to do.

Ellis. Ellis's work on top-down causation argues that physics is not causally closed at the microphysical level and that higher-level structures exert genuine causal influence [7]. My argument is about the closure of $\mathcal{E}$ as a whole rather than closure between its levels, but the two share an opponent, which is the assumption that the lower level is self-sufficient.

Tegmark. The mathematical universe hypothesis identifies physical existence with mathematical structure [23]. I discussed it in Section 5. It is the cleanest illustration of why identifying $\mathcal{E}$ with formal structure sharpens the grounding problem rather than dissolving it.

8 What would defeat this argument

The core claim is a conditional result in logic, not an empirical hypothesis. No measurement bears on it directly. The claim would be defeated by argument, and there are four places to attack.

Deny that existence exhibits formal structure. If the regularities in $\mathcal{E}$ are not lawful, not consistent, or not mathematically expressible, premise one fails and the theorem has nothing to apply to. This is a coherent position and it is difficult to hold, since the practice of physics presupposes what it denies.

Deny consistency. If $\mathcal{E}$ is inconsistent, premise two fails. Gödel's theorem is silent about inconsistent systems, which prove everything including their own consistency. The cost is high: an inconsistent existence has no reason to exhibit the stable regularity we observe, and the stability then wants an account of its own. I regard this as relocating the problem rather than solving it, but someone willing to pay the cost has a genuine exit from this route only. The other three constraints of Section 4.4 would still hold, and with them most of the eliminating.

Find a third horn in the disjunction. Section 3.1 claims that $\mathcal{E}$ either is a formal system or produces formal structure from a non-formal substrate. A demonstration that the disjunction is not exhaustive, or that the second horn carries no explanatory burden, would break the argument at its widest point. This is where I would attack it. But again, breaking the Gödelian route leaves the other three constraints untouched.

Exhibit a ground inside $\mathcal{E}$ that meets all four conditions. Section 5 surveys the candidates I know of. The survey is not a proof of exhaustiveness, and a new candidate satisfying transcendence, productive capacity, freedom from determinate properties and the sustaining of order would refute the elimination directly. I invite the attempt, since a successful one would teach me more than agreement.

It also helps to say what would *not* defeat this argument. People point to things science cannot work out, and three of those come up here. Chaos is one. Nobody forecasts the weather a month ahead, because a tiny error in today's measurements grows into a huge one. The weather still follows its laws exactly, so the issue is our measuring. Intractability is another. A computer could work the answer out, but it would need more time or memory than we can ever give it. Undecidability is the third. There the laws fix the answer and no recipe finds it in every case, so what is missing is a method.

All three are limits on what we can find out. My claim is about whether lawful structure can hold itself up, which is a different question. I do not want support from evidence that answers a different question.

The results in Section 3 have one purpose. They show that the mathematics of physics is rich enough for Gödel's theorems to apply to it.

9 Objections

Franzén's objection. Franzén catalogued the misuse of Gödel's theorems outside mathematics and warned specifically against arguments that extend incompleteness to reality as a whole [10]. The warning is deserved. Most such arguments equivocate on "system" or treat the theorems as a general principle that everything is incomplete, which they are not.

My argument does not require that extension. It requires that formal systems exist within $\mathcal{E}$, which is not contentious, and that they cannot self-ground, which is Gödel's theorem applied to exactly the objects it was proved about. The disjunctive form in Section 3.1 was constructed so that nothing turns on whether $\mathcal{E}$ as a whole counts as a formal system. If it does, the theorem applies. If it does not, the emergence of formal structure needs an account instead. Franzén's stricture bites on arguments that need the first horn and assume it without justification. It does not bite on an argument that survives either horn.

Consistency is an artefact of observation. One might hold that we perceive consistency because inconsistent regions are unobservable, or that our reasoning selects for it. Grant this and the grounding requirement changes form rather than disappearing. An inconsistent existence that nonetheless supports stable observers, reliable mathematics and repeatable physics is more puzzling than a consistent one, and the stability still wants an account. The properties still want explaining.

Laws are descriptions, not prescriptions. On a Humean reading, laws summarise regularities rather than enforce them, so there is no formal system doing any governing. But the regularities are what my argument needs. Whether we call the mathematics a description or a governor, it is exact, it is consistent, and it holds where nobody is describing anything. Demoting laws to summaries leaves untouched the question why the summarised behaviour has the structure that makes summary possible.

The argument is self-undermining. If no formal system grounds itself, and this argument is a formal argument inside $\mathcal{E}$, does it undermine its own conclusion?

It does not, and the reason is instructive. The argument does not claim to prove its own consistency. It claims that a consistent system of the relevant kind cannot prove its consistency internally, which is a claim proved in ordinary mathematics about mathematical objects. That the conclusion also applies to the argument is expected, and it limits what the argument can go on to establish. Section 6 stops where it does for exactly this reason. An argument inside $\mathcal{E}$ can establish that a ground is required. It cannot, from inside, establish what the ground is, and I have tried not to claim otherwise.

10 Conclusion

Existence exhibits lawful, consistent, mathematically exact structure. That structure either is a formal system or produces one, and by Gödel's second theorem neither can account for its own consistency from within. The account must come from outside. This requirement is what I have called trans-existential grounding.

Three further routes bear on the same requirement without using the theorem, and I have been careful not to claim more for them than they give. The exhaustive why is sufficient reason in procedural dress and does not count separately. The property regress does, since it runs on a dependency relation rather than on any demand for explanation, and its trilemma leaves the property-free terminus as the only exit that is neither an infinite descent nor an arbitrary stop. So the argument has three starting points, and rejecting the most contested of them, the Gödelian one, leaves the other two intact.

Each route also fixes one constraint on the ground: outside, necessary, property-free, needing no further why. Their agreement is a composite description as much as a corroboration, and the constraints are what do the eliminating that follows.

Candidates inside existence fail without exception. Each is part of what needs grounding, or possesses the determinate character that would make it part of it.

What is left is a set of conditions on any adequate ground, not the ground itself. I have argued only for the requirement, which seems to me the part that can be established rather than proposed. It is a smaller conclusion than the question invites, and I think it is the one the argument supports.

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