Gödel, Against the Final Theory

Geir Isene
isene.com

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This note belongs to the first paper in this series, *Why Existence Cannot Ground Itself* [6]. There, existence is everything that has properties, written $\mathcal{E}$. *The premise* says that whatever is one way and not another must have an account.

1 The argument

"Physics will one day reach the final theory, and that theory will account for itself. Then the chain of causes ends inside existence after all."

It does not. Written out, a final theory is a formal system with arithmetic inside it, and no such system can establish that it is free of contradiction.

The arithmetic comes with the physics. The wave equation can turn numbers a computer handles into answers no computer can reach [9]. Whether a set of tiles can cover a floor has no general answer [1]. A quantum material can hide a question no method decides [2]. Mathematics this rich contains arithmetic. And existence is consistent: no contradiction has ever shown up in physics.

Gödel showed in 1931 that no consistent system with arithmetic inside can establish its own consistency [5]. A stronger system can establish it for a weaker one, and still cannot do the same for itself. Add a third, and so on. Take the whole ladder together. If it amounts to one more such system, it cannot vouch for itself. The ladder never finishes: the full truth of arithmetic, which would finish it, is no formal system [10]. If it is no system at all, it still asks for an account.

The same point with machines. Picture a machine reading the laws, spelled out one by one, hunting for a contradiction. To call the laws consistent is to say the hunt never ends. Gödel's second theorem, put this way: no system can show that its own hunt never ends. A stronger system can show it for a weaker one, and then its own hunt stays unproven. The ladder once more, and Turing's halting problem is this limit seen from the machine's side [11].

So the final theory, if we ever write it, leaves its own consistency unaccounted for, and hence it is not a theory of *everything*. The first paper's argument applies to that consistency [6]. The exact statements and their proofs are in Section 2.

Say instead that existence is no system, and only contains systems. Then something with no formal character keeps producing formal character: arithmetic, logic, exact laws, at every scale. That production asks for an account, so the problem is renamed and not removed.

Franzén's warning. Franzén is right that the theorems get stretched past breaking [4]. This section stretches nothing. Formal systems exist inside the world, and they cannot vouch for themselves. That is Gödel used on the very things he proved it for. These limits say nothing about minds, unlike the arguments of Lucas and Penrose [7, 8]; they are about systems. Undecidability closes one escape, the hope of a self-accounting theory, and supplies no cause.

Nothing in the first paper depends on this note.

2 The formal statements and proofs

Here Section 1 gets its exact form. Written out, a final theory is a formal system that contains arithmetic, and existence is consistent.

Definition 1 (Certificate)**.** Say a system T *certifies* a system S when T proves $\mathrm{Con}(S)$. The certificate counts only when T is consistent itself, since an inconsistent system proves anything.

Theorem 1 (The certification ladder never closes)**.** *Take a consistent formal system S that contains arithmetic, with axioms a procedure can list. Then:*

1. *S cannot certify itself.*
2. *Whatever certifies S proves something beyond the reach of S. And if that certifier is another such system, it cannot certify itself either.*
3. *Climb the ladder: start from S and keep adding "the system so far is consistent" as a new step. If each step is consistent, the ladder as a whole is another such system, and so it cannot certify itself either.*
4. *Any other such chain, each system containing and certifying the one before it, ends the same way. Gathered into one, it is either another such system, and clause (1) applies, or no procedure can list its axioms, in which case it is not a formal system at all.*

Proof. Clause (1) is Gödel's second incompleteness theorem [5]. For clause (2): if a part of S proved $\mathrm{Con}(S)$, that proof would already live inside S, which (1) forbids. And the certifier, being another such system, is covered by (1) as well.

For clause (3): the procedure that lists the axioms of each step also lists the axioms of the whole ladder. Every step contains the ones before it. A contradiction needs only finitely many axioms, so it would already sit inside one step. Hence the ladder is consistent whenever every step is, and (1) applies to it.

For clause (4): gathered into one, the chain is consistent by the same argument, and arithmetic is inside it. Whether (1) applies turns on a single question: can a procedure list its axioms? □

Remark 1. One can push the ladder beyond the finite rungs [12, 3]. It gains nothing. Any stage a procedure can describe is one more formal system, with a next rung above it. Only the whole of arithmetical truth would end the climb, and Tarski's theorem says that is no formal system [10]. It is something else in need of an account.

Theorem 2 (The grounding requirement)**.** *Take S to be a consistent formal system inside $\mathcal{E}$ with arithmetic inside it. Suppose the consistency of S has an account, meaning it is no brute fact. Then:*

1. *The account is given by no formal system.*
2. *If no candidate inside $\mathcal{E}$ supplies it, then it comes from outside $\mathcal{E}$.*

Proof. (1) Theorem 1 says every formal certificate comes from a further system, which then needs one of its own. A chain of certificates that never lands certifies nothing. The first paper gives the general reason [6]: an endless chain is a brute fact, since the whole chain has no account. Hence the account stops at something that is no formal system.

(2) Two candidates remain inside $\mathcal{E}$. The first is a structure without being a system: the natural numbers, or mathematical truth as a whole. The first paper deals with it among the numbers and forms [6]. The second is the brute fact, ruled out by the premise. □

Those two points hand us the system. The first makes the lawful structure of $\mathcal{E}$ a formal system containing arithmetic. The second makes it consistent. The system lies inside $\mathcal{E}$ by the definition of existence. Theorem 2 then applies to it.

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